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RESIDENCE TIME DISTRIBUTION (RTD) ASSOCIATED WITH STATISTICAL WEIGHTED MOMENTS TO FIT MATHEMATICAL MODELS USING IN INDUSTRIAL SYSTEMS DIAGNOSIS

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ABSTRACT

This work presents a computational, called MOMENTS, code developed to be used in process control to determine a characteristic transfer function to industrial units when radiotracer techniques were been applied to study the unit´s performance. The methodology is based on the measuring the residence time distribution function (RTD) and calculate the first and second temporal moments of the tracer data obtained by two scintillators detectors NaI positioned to register a complete tracer movement inside the unit. Non linear regression technique has been used to fit various mathematical models and a statistical test was used to select the best result to the transfer function. Using the code MOMENTS, twelve different models can be used to fit a curve and calculate technical parameters to the unit.

1. INTRODUCTION

For the industry, with the increasing price of materials and in the energy cost, there is a great interest in minimizing costs and any possibility to evaluate the performance of an installation to prevent a fault is important.

Mathematical modeling of an industrial processes has became one of the most important tool in process control and analysis of a system because it make possible to built a model to describe, observe and control the behavior of the process but only in specials cases a mathematical model will be able to describe all the details of a process.

Generally a model can be specified from basic principles, but if there is little information about the system and a model must be constructing using technical data measure direct from the unit. This methodology is called “parameter estimation” problem and to construct a model depend on the type of mathematical formulation choose. The greatest limitation of this procedure is that is practically impossible to describe a real system with simple mathematical formalism, so it is usual to use simplifications to the model formulation to approximate an accurate mathematical model.

The most use method to obtain the system transfer function and to fit a mathematical model is the impulse/response technique where the system is stimulated by an impulse signal (most common is rapid imperfect pulse) and the output signal is measure as function of time.

2. METHODOLOGY

Any system can be represented by a mathematical operator and the process under study can be represented by functions defined a convolution equation:

$$Y(t) = H(t) \otimes X(t) \quad \rightarrow \quad Y(t) = \int_0^t X(u).H(t-u)du \tag{1}$$

Where:
X (t) - Input function (tracer injection)
H (t) - Transfer function (depends on system internal processes)
Y (t) - Output function (system response to input stimulus)

Changes in the tracer concentration, as it moves through the unit, are caused mainly by the action of different physical-chemical process that taking place inside the unit but the response, Y(t), depends on both the action of the system function, H (t), as due to the tracer injection process, X (t).

2.1. Mathematical Formalism

In the experimental tests, the radiotracer has to be injected in a section located before the measure points to ensure the complete mixing in the system flow. To measure the tracer concentration two scintillators detectors NaI were used one in the entrance and the second in the unit´s exit. Considering the count rates registered by each detector, the convolution equation (1) can be written as [1,2]:

$$C_{out}(t) = \int_0^t C_{in}(t).H(t-u)du \tag{2}$$

Where: C_{out} (t) - normalized curve registered by the detector in the output
C_{in} (t) – normalized curve registered by the detector in the entrance

Applying the Laplace transformation in the equation (2) we have a simple equation in Laplace space:

$$\mathrm{Y(S)} = \mathrm{H(S)} * \mathrm{X(S)} \quad \Rightarrow \quad \mathrm{H(S)} = \frac{\mathrm{Y(S)}}{\mathrm{X(S)}} = \frac{\mathrm{C_{out}(S)}}{\mathrm{C_{in}(S)}} \tag{3}$$

Where: H (S) - Transfer Function in Laplace space
C_{in} (S) - Input function in Laplace space
C_{out} (S) - Out function in Laplace space

Deriving equation (2) with respect to S (two times), we have:

$$\frac{H^{'}(S)}{H(S)} = \frac{C_{out}^{'}(S)}{C_{out}(S)} + \frac{C_{in}^{'}(S)}{C_{in}(S)} \tag{4}$$

and

$$\left[\frac{H^{''}(S)}{H(S)} - \left(\frac{H^{'}(S)}{H(S)}\right)^2\right] = \left[\frac{C_{out}^{''}(S)}{C_{out}(S)} - \left(\frac{C_{out}^{'}(S)}{C_{out}(S)}\right)\right] - \left[\frac{C_{in}^{''}(S)}{C_{in}(S)} - \left(\frac{C_{in}^{'}(S)}{C_{in}(S)}\right)\right] \tag{5}$$

Associating this formalism to the statistical weight moments [1] is possible to describe the unit by estimating parameters to a mathematical model. Using the k-order weight moment for any statistical distribution and considering the normalized function for the signal registered by the first detector, $C_{in}(t)$, and for the second $C_{out}(t)$, we have:

$$H(S) = \frac{m_{out}^{0}(S)}{m_{in}^{0}(S)} \tag{9}$$

$$\frac{H^{'}(S)}{H(S)} = \frac{m_{in}^{1}(S)}{m_{in}^{0}(S)} - \frac{m_{out}^{1}(S)}{m_{out}^{0}(S)} \tag{10}$$

$$\left[\frac{H^{''}(S)}{H(S)} - \left(\frac{H^{'}(S)}{H(S)}\right)^2\right] = \left[\frac{m_{out}^{2}(S)}{m_{out}^{0}(S)} - \left(\frac{m_{out}^{1}(S)}{m_{out}^{0}(S)}\right)^2\right] - \left[\frac{m_{in}^{2}(S)}{m_{in}^{0}(S)} - \left(\frac{m_{in}^{1}(S)}{m_{in}^{0}(S)}\right)^2\right] \tag{11}$$

To the parameters estimation first we have to compute the moments from the experimental data and the then calculate de moments using the theoretical expression for a special model. Using a non linear square least square method the parameters can be fit and the transfer function can be easily constructed without necessity of inverting the Laplace transform.

2.2. Transference Functions for Theoretical Flow Models

Flow models are mathematical description of flow and mixing inside the unit and they give a macroscopic description of the main process are occurring. They are useful for understand the mass transport inside the unit and analysis of and analysis for malfunction of the system. In MOMENTS is possible to choose the models [3,4,]:

- ELEMENTARY MODELS: Plug Flow; Perfect Mixer; Axial dispersed plug flow.

- MODELS WITH UNITS IN SERIES AND PARALLEL: N perfect mixers (identical) in series; N perfect mixers (identical) in series in parallel with M mixers (identical) in series; 02 different mixers in series; 02 different mixers in parallel; Plug flow in parallel with 02 different mixers in series; Plug flow in parallel with N mixers (identical) in series; Axial dispersed plug flow in parallel with plug flow; Axial dispersed plug flow in parallel with N mixers (identical) in series.

- MODELS WITH “BY PASS”: Plug Flow; Perfect Mixer; Axial dispersed plug flow; Axial dispersed plug flow in series with perfect mixer.

- MODELS WITH RECIRCULATION: Perfect Mixer; Axial dispersed plug flow; Perfect Mixer and plug flow in series; 02 different mixers in series; 02 different mixers in parallel; N mixers (identical) in series; Perfect mixer with recirculation by plug flow; N perfect mixers (identical) in series with recirculation by M mixers (identical) in series.

3. EXPERIMENTAL TESTS

To evaluate the program MOMENTS artificial data were used to simulate different operational conditions for real units (mixers, flow in pipes, flow in packed columns, dead zone, and recirculation in tanks) and compare the fitted parameters with the simulated ones for each mathematical model.

The results showed that is possible to fit technical parameters to a theoretical equation with great accuracy (errors around 0.1%). In this work we try to fit an equation to real cases.

3.1. Transfer Functions for Water Flow in Pipe

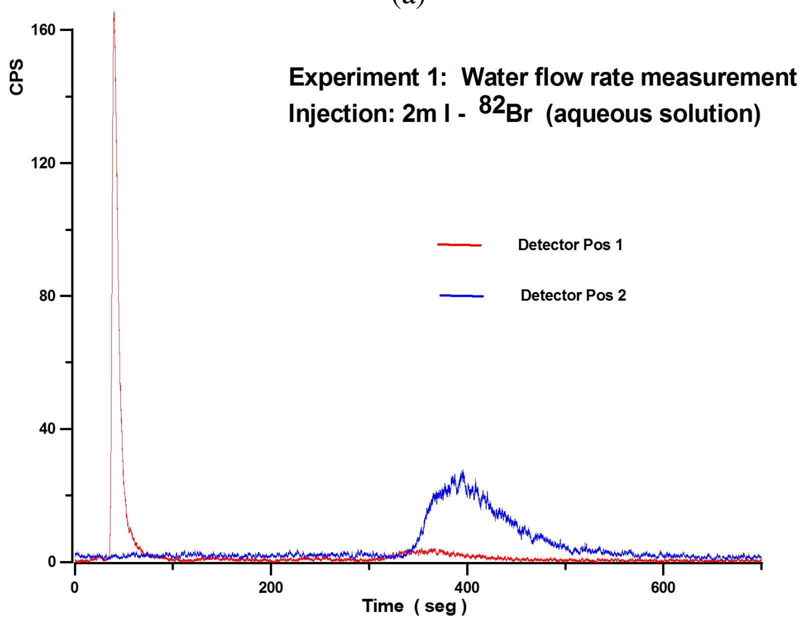
The first situation was the water flow rate measurement in pipe using ^{82}Br as radiotracer; this was chosen because it is one of the most important tracer applications in industries. In this experiment a fast pulse of ^{82}Br was injected (20 ml of KBr aqueous solution); two scintillators NaI detectors were used to registered tracer movement, one was located 6.0m far from the injection point and the second was 30.0 m far from the first one.

The intention is to use a mathematical model to fit not only the transient time but both to study how was the displacement and if exist any flow perturbation as the existence of blockage or channeling inside the pipe. The results for the 0-order moment, equation 9,are shown in Figure 1a and Figure 1b.

Figure 1a shows the tracer movement during the experiment. In the first position, the tracer is concentrated and its passage through the region of detector 1 show a typical curve but in the second position the detector 2 registered a curve with a tail.

Using different models is possible to study this situation , in this case it may be simply due to diffusion is the only phenomenon present. Any perturbation the signal recorded by the detectors can indicate the existence of problems, for example an partial obstruction inside the duct.

(a)



(b)

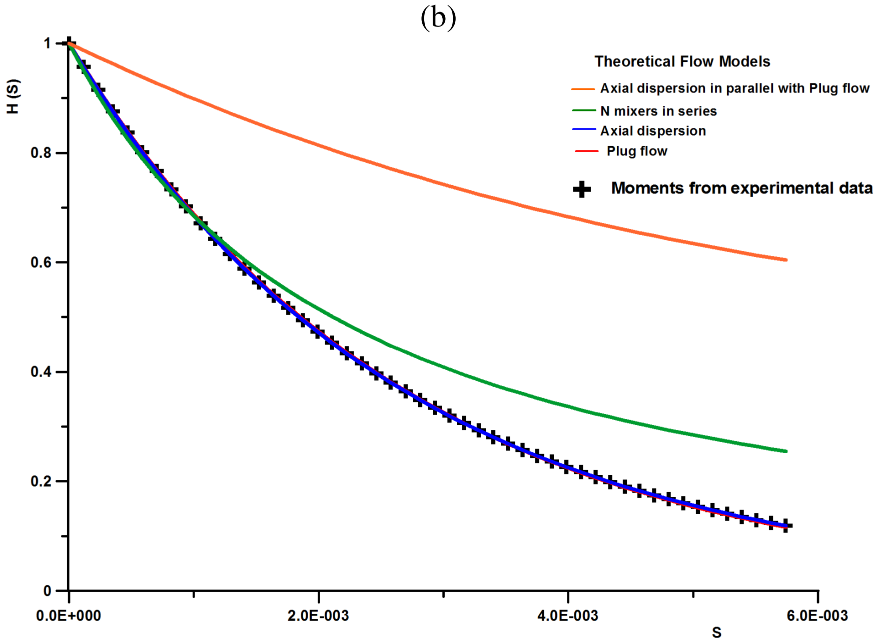


Figure 1. (a) Tracer movement inside the pipe; (b) Fitting models to H(S) for 0^{th}-moment

We tried to fit: a plug flow; an axial dispersion plug flow; an identical mixers in series and an axial dispersion in parallel with plug flow models. If there is something inside the pipe modifies the flow profile a model with parallel sub-system will be better fitted. In Table 1 we have the fitted parameters and de variance for each fit.

Table 1. Fitted Parameters using MOMENTS to water flow in pipe

Model	Parameters	Fitted Value	Variance S^2
Plug Flow	τ_P	(374,17 ±0.66) s	.148 E-02
Axial Dispersion	τ_D	(379,74 ±0.42) s	.251 E-03
	P	(899,98 ±0.04)	
Mixers in Series	**N**	(1.2 ± 0.4)	0.36 E 0
	τ_M	(371,72 ±0.68) s	
Axial Dispersion in parallel with Plug Flow	τ_D	(255.68±3.83) s	7.11 E 0
	P	(911.43±4.5)	
	τ_P	(151.67 ± 1.37)	
	α	0.80 ± 0.02	

The best model is Axial dispersion (S^2= 2.51e-1) showing the fluid displacement is, as expected, normal and with axial dispersion phenomenon mainly responsible for the tracer curve enlargement in position 2. When we tried to fit a model with sub-system in parallel to study the possibility of an abnormal operational situation se fin variance S2 was the worst for all adjusts.

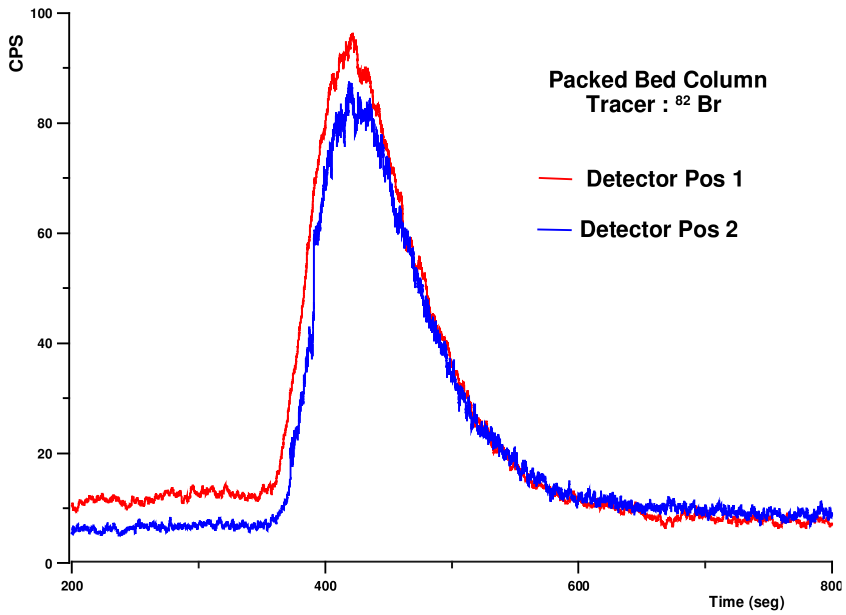
3.2. Transfer Functions for a packed bed column

In industry, a packed column is widely applied to perform separation processes, such as absorption, stripping, and distillation. The column can be filled with random dumped packing or structured packing sections and after a long time operation serious problems as channeling, obstruct and blockage. Using MOMENTS we can study the flow profile and verify the existence of operational problems.

In our laboratory we have a small column, made in PVC with 3.0 cm long and with internal diameter of 6.0 cm filled with small glass balls (0.5 cm in diameter) and water passed through the column with 3.0 l/s. The distance between the scintillators detectors was 9.0 cm. Figure2a shows the response of an ^{82}Br pulse (injected far from the column), the mean residence time for this system is τ = (5.125 ± 0.057) s and the curves are typical for this situation. Analyzing the residence time distribution curves is impossible to ensure there is no problem inside the column.

Figure2b shows mathematical flow models fitted using MOMENS for1th- moment, equation (10) and the best fit was using axial dispersion plug flow (S^2 = 1.28 E-3) and this is a strong evidence that the column is operating in a normal situation with water only passing through the column only dispersed by action of axial dispersion coefficient D_{ax}.

(a)



(b)

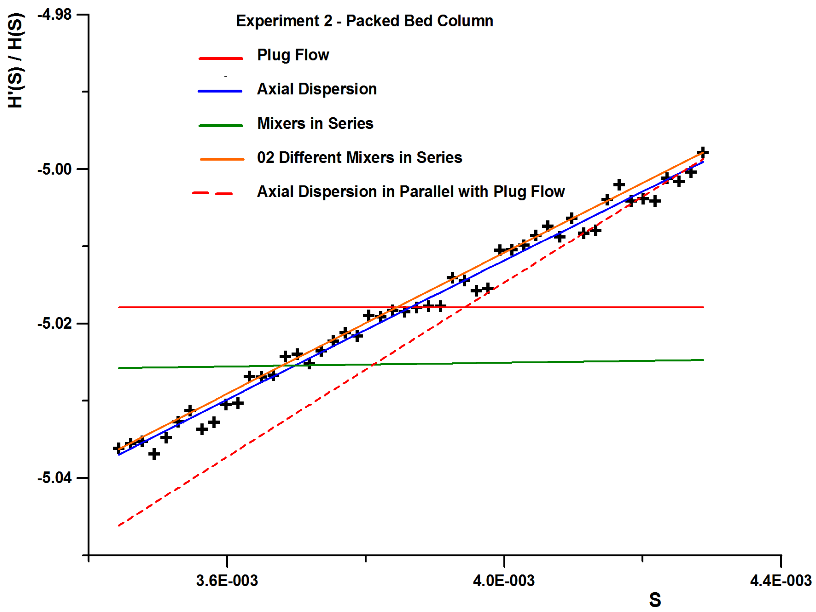


Figure 2. (a) Tracer movement inside the column; (b) Fitting models to H'(s)/H(S) for 1^{th}-moment

In table2 we have the parameters fitted for five different mathematical models. Considering the variance S^2, two models are possible to use to describe the flow profile inside the column: axial dispersion plug flow and 02 different mixers is parallel.

Models in parallel reveal the possible existence of two channels for the fluid displacement inside the column, ie, the existence of two transport areas with different characteristics. In Table 1, considering two different mixers model, the fitted parameter α, the fraction of flow in mixer 1 branch demonstrate that the flow contribution passing through mixer1 (τ_1 = 1.09 s) is small. Again if we compare another parallel model, axial dispersion and plug flow, we note there is a possibility of a small contribution in the plug flow branch.

Table 2. Fitted Parameters using MOMENTS packet bed column

Model	Parameters	Fitted Value	Variance S^2
Plug Flow	τ_P	5.02 s	0.50 E+01
Axial Dispersion	τ_D	5.03 s	1.28 E-03
	P	1.08	
Mixers in Series	**N**	20.56	0.15 E +01
	τ_M	5.03	
02 Different Mixers in Series	τ_1	1.09	1.07 E -02
	τ_2	7.64	
	α	0.37	
Axial Dispersion in parallel with Plug Flow	τ_D	5.82	3.08 E -2
	P	0.998	
	τ_P	1.7	
	α	0.706	

3. CONCLUSIONS

The transfer function to a real system can be evaluate using the program MOMENTS from experimental data registered by two scintillators detectors after a single radiotracer pulse (imperfect fast pulse) injected in a point located before the unit. The methodology the methodology is based on weighted statistical moments calculated from the data of the curves of normalized count rate.

As the results show is possible do study to fit an arbitrary mathematical model and then postulate information about the situation inside the unit. The program MOMENTS is now working in our laboratory with 22 different models and we are developing more complex models as models with internal baking mixing, backflow cell models, stochastic mixing models for chemical reactors.

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